

Friday 17 June 2016 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The differential equation

$$4 \frac{d^2x}{dt^2} + 8 \frac{dx}{dt} + 3x = f(t)$$

is to be solved.

Consider first the case where $f(t) = 4e^{0.5t}$.

(i) Find the general solution for x . [7]

(ii) Given that when $t = 0$, $x = 6$ and $\frac{dx}{dt} = -4$, find the particular solution for x . [4]

(iii) Given that x has a minimum value, find the value of t for which this minimum occurs. [4]

Now consider the case where $f(t) = 4e^{-0.5t}$.

(iv) Given that when $t = 0$, $x = 6$ and $\frac{dx}{dt} = -4$, find the particular solution for x . [9]

2 (a) The differential equation

$$x \frac{dy}{dx} - 3y = x^5 \cos x$$

is to be solved.

(i) Find the general solution for y in terms of x . [8]

(ii) Find the particular solution for which $y = 0$ when $x = \frac{1}{2}\pi$. [2]

(b) Now consider the differential equation $\sec x \frac{dy}{dx} - 3y^2 = 0$.

(i) Find the particular solution for which $y = 1$ when $x = \frac{1}{2}\pi$. [7]

(ii) Show that the maximum value of y is 1. [2]

(c) Now consider the differential equation $\sec x \frac{dy}{dx} - 3y^2 = x$, where $y = 0$ when $x = 1$.

This is to be solved numerically using Euler's method. The algorithm is given by

$$x_{r+1} = x_r + h, \quad y_{r+1} = y_r + hy'_r.$$

Use a step length of 0.01 to estimate y when $x = 1.02$. [5]

- 3 A parachutist of mass 90 kg falls vertically from a stationary helicopter. When he is x m below the helicopter, his velocity is v m s⁻¹. The forces acting on the parachutist are his weight and a resistive force of magnitude $0.36v^2$ N.

(i) Show that his motion can be modelled by the differential equation

$$v \frac{dv}{dx} = 9.8 - 0.004v^2. \quad [2]$$

(ii) Solve this differential equation to show that

$$v^2 = 2450(1 - e^{-0.008x}). \quad [6]$$

(iii) Sketch the graph of v against x . [2]

(iv) Find how far the parachutist has fallen when his speed has reached 48 m s⁻¹. [2]

The parachutist opens his parachute when his speed is 48 m s⁻¹. The forces acting on him now are his weight and a resistive force of magnitude $72v$ N.

(v) Find an expression for v in terms of t , where t is the time in seconds that has elapsed since the parachute was opened. [8]

(vi) Find the distance that the parachutist falls during the first 5 seconds after his parachute has opened. [4]

- 4 The simultaneous differential equations

$$\frac{dx}{dt} = x - y + 3 \cos t$$

$$\frac{dy}{dt} = 5x - y - 12 \sin t$$

are to be solved for $t \geq 0$, where t denotes time.

(i) Eliminate y to obtain a second order differential equation for x in terms of t . Hence find the general solution for x . [12]

(ii) Find the corresponding general solution for y . [3]

When $t = 0$, $y = 0$ and $\frac{dy}{dt} = 5$.

(iii) Find the particular solutions for x and y . [4]

(iv) Find the time that elapses between the first two occasions on which $x = y$. [5]

END OF QUESTION PAPER

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Candidate forename		Candidate surname	
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Centre number						Candidate number				
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1 (ii)	

1 (iii)	

1 (iv)	(continued)

2(a)(i)	
2(a)(ii)	

2 (b)(i)	

<p>3 (ii)</p>	<p>(continued)</p>
<p>3 (iii)</p>	
<p>3 (iv)</p>	

3(vi)	

4 (i)	(continued)
4 (ii)	

4(ii)	

4 (iv)	



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GCE

Mathematics (MEI)

Unit **4758**: Differential Equations

Advanced GCE

Mark Scheme for June 2016

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It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.



All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
 and 	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
MO M1	Method mark awarded 0, 1
AO A1	Accuracy mark awarded 0, 1
B0 B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

1. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep **' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.

It should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for g . E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

- i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

1. (i)	<p>AE: $4m^2 + 8m + 3 = 0$</p> $m = -1 \pm \frac{1}{2} = -\frac{1}{2}, -\frac{3}{2}$ <p>CF: $x = Ae^{-\frac{1}{2}t} + Be^{-\frac{3}{2}t}$</p> <p>PI: $x = ke^{\frac{1}{2}t}$</p> $k = \frac{1}{2}$ $x = Ae^{-\frac{1}{2}t} + Be^{-\frac{3}{2}t} + \frac{1}{2}e^{\frac{1}{2}t}$	<p>M1</p> <p>A1</p> <p>F1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>F1</p> <p>[7]</p>	<p>Differentiate and substitute to find k</p> <p>PI + CF with 2 arbitrary constants</p>	
(ii)	<p>$t = 0, x = 6 \quad 6 = A + B + \frac{1}{2}$</p> <p>$t = 0, \frac{dx}{dt} = -4 \quad A + 3B = \frac{17}{2}$</p> <p>$A = 4, B = \frac{3}{2} \quad x = 4e^{-\frac{1}{2}t} + \frac{3}{2}e^{-\frac{3}{2}t} + \frac{1}{2}e^{\frac{1}{2}t}$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Use condition</p> <p>Differentiate and use condition</p> <p>Solve</p>	
(iii)	<p>Minimum when $-2e^{-\frac{1}{2}t} - \frac{9}{4}e^{-\frac{3}{2}t} + \frac{1}{4}e^{\frac{1}{2}t} = 0$</p> <p>$e^{2t} - 8e^t - 9 = 0$</p> <p>$(e^t + 1)(e^t - 9) = 0$</p> <p>$t = \ln 9 = 2 \ln 3$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Differentiate and equate to 0</p> <p>Multiply through by $e^{\frac{3}{2}t}$</p> <p>Solve</p> <p>cao o.e (2.19722...)</p>	

(iv)	<p>CF: $x = Ce^{-\frac{1}{2}t} + De^{-\frac{3}{2}t}$</p> <p>PI: $x = Qte^{-0.5t}$</p> <p>$\dot{x} = Q(1 - 0.5t)e^{-0.5t}$; $\ddot{x} = Q(-1 + 0.25te^{-0.5t})$</p> <p>$4Q = 4$</p> <p>$Q = 1$</p> <p>$x = te^{-0.5t} + Ce^{-\frac{1}{2}t} + De^{-\frac{3}{2}t}$</p> <p>$\dot{x} = -\frac{1}{2}Ce^{-\frac{1}{2}t} - \frac{3}{2}De^{-\frac{3}{2}t} + e^{-0.5t}(1 - 0.5t)$</p> <p>$t = 0, x = 6$: $C + D = 6$</p> <p>$t = 0, \frac{dx}{dt} = -4$: $C + 3D = 10$</p> <p>$C = 4, D = 2$</p> <p>$x = te^{-0.5t} + 4e^{-\frac{1}{2}t} + 2e^{-\frac{3}{2}t}$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[9]</p>	<p>FT from (i)</p> <p>Differentiate using product rule and substitute</p> <p>Compare coefficients and solve</p> <p>cao</p> <p>Differentiate using product rule</p> <p>Use condition correctly</p> <p>Use condition correctly (independent of use of product rule) and solve</p> <p>cao</p>	
2. (a)(i)	<p>Divide through by x</p> <p>IF: $e^{\int -\frac{3}{x} dx}$</p> <p>$= e^{-3 \ln x}$</p> <p>$= x^{-3}$</p> <p>$\frac{d}{dx}(yx^{-3}) = x \cos x$</p> <p>RHS = $x \sin x + \cos x + c$</p> <p>$y = x^3(x \sin x + \cos x + c)$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>F1</p> <p>[8]</p>	<p>Both sides</p> <p>Attempt to find integrating factor</p> <p>Integrate and simplify log term</p> <p>Multiply both sides by IF and express LHS as differential o.e.</p> <p>Integrate by parts</p> <p>Including + c</p> <p>Divide through by their integrating factor</p>	

<p>(ii)</p> <p>(b)(i)</p>	$y = 0 \text{ when } x = \frac{1}{2}\pi : 0 = \left(\frac{\pi}{2}\right)^3 \left(\frac{\pi}{2} + c\right)$ $c = -\frac{\pi}{2} : y = x^3 \left(x \sin x + \cos x - \frac{\pi}{2}\right)$ $\frac{dy}{y^2} = \frac{3dx}{\sec x}$ $-\frac{1}{y} = \int 3 \cos x \, dx$ $= 3 \sin x + c$ $y = 1 \text{ when } x = \frac{1}{2}\pi : c = -4$ $y = \frac{1}{4 - 3 \sin x}$	<p>M1</p> <p>A1</p> <p>[2]</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>Use condition</p> <p>cao</p> <p>Separate the variables</p> <p>Attempt to integrate</p> <p>LHS</p> <p>Integral of RHS including + c</p> <p>Use condition</p> <p>Make y the subject</p> <p>cao Final answer</p>																	
<p>(ii)</p>	<p>Max value when $\sin x = 1$</p> <p>Max value = $1/(4-3) = 1$ AG</p>	<p>M1</p> <p>E1</p> <p>[2]</p>	<p>Correct statement for their solution to (b)(i)</p> <p>Or any other valid method e.g. differentiation, second derivative not required</p> <p>Correctly shown; $1/(4-3)$ must be seen</p> <p>SC2 for stating BOTH max value = 1 and min value = $1/7$</p>																	
<p>(c)</p>	$\frac{dy}{dx} = (3y^2 + x) \cos x$ <table border="1" data-bbox="353 1114 954 1264"> <thead> <tr> <th>x</th> <th>y</th> <th>y'</th> <th>hy'</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>0.5403023</td> <td>0.005403023</td> </tr> <tr> <td>1.01</td> <td>0.005403023</td> <td>0.5372259</td> <td>0.005372259</td> </tr> <tr> <td>1.02</td> <td>0.010775282</td> <td></td> <td></td> </tr> </tbody> </table> <p>0.0108</p>	x	y	y'	hy'	1	0	0.5403023	0.005403023	1.01	0.005403023	0.5372259	0.005372259	1.02	0.010775282			<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>May be implied by correct values</p> <p>Use algorithm</p> <p>$y(1.01) = 0.005403 \dots$ Agreement to 3 sf</p> <p>$y'(1.01) = 0.5372259 \dots$ Agreement to 3 sf</p> <p>0.011 or better</p>	
x	y	y'	hy'																	
1	0	0.5403023	0.005403023																	
1.01	0.005403023	0.5372259	0.005372259																	
1.02	0.010775282																			

3.	(i)	$90v \frac{dv}{dx} = 90g - 0.36v^2$ $v \frac{dv}{dx} = 9.8 - 0.004v^2.$	M1 E1 [2]	Use N2L, 3 terms, allow sign errors, allow any form for accn, including <i>a</i>	
	(ii)	$\frac{v dv}{9.8 - 0.004v^2} = dx$ $-\frac{1}{0.008} \ln 9.8 - 0.004v^2 = x + A$ <p>Use $v = 0, x = 0$: $A = -\frac{1}{0.008} \ln 9.8$</p> $v^2 = 2450(1 - e^{-0.008x})$	M1* A1 A1 M1dep* M1dep* E1 [6]	Separate variables Integrate to obtain LHS RHS including constant on one side Use condition Rearrange	
	(iii)	Increasing graph through (0, 0) Asymptote $v = \sqrt{2450}$	B1 B1 [2]	Allow 49.5	
	(iv)	353 m	M1A1 [2]	Accept 2.s.f.	
	(v)	$90 \frac{dv}{dt} = 90g - 72v$ $\frac{dv}{dt} = g - 0.8v$ $\frac{dv}{9.8 - 0.8v} = dt$ $-\frac{1}{0.8} \ln 9.8 - 0.8v = t + B$	M1 A1 M1 A1 A1	Use N2L, 3 terms, allow sign errors Separate variables Integrate to obtain LHS RHS, including constant on one side	

	<p>Use $t = 0, v = 48$: $B = -\frac{1}{0.8} \ln 28.6$</p> $v = 12.25 + 35.75e^{-0.8t}$ <p>Alternative method 1:</p> $90 \frac{dv}{dt} = 90g - 72v$ $\frac{dv}{dt} = g - 0.8v$ <p>Integrating factor: $e^{0.8t}$</p> $\frac{d}{dt}(ve^{0.8t}) = ge^{0.8t}$ $ve^{0.8t} = 12.25e^{0.8t} + A$ $v = 12.25 + Ae^{-0.8t}$ <p>Use $t = 0, v = 48$: $A = \frac{143}{4} = 35.75$</p> $v = 12.25 + 35.75e^{-0.8t}$ <p>Alternative method 2:</p> $90 \frac{dv}{dt} = 90g - 72v$ $\frac{dv}{dt} = g - 0.8v$ <p>CF: $Ae^{-0.8t}$</p> <p>PI: $v = 12.25$</p> <p>GS: $v = Ae^{-0.8t} + 12.25$</p> <p>Use $t = 0, v = 48$: $A = 35.75$</p> $v = 12.25 + 35.75e^{-0.8t}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[8]</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[8]</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[8]</p>	<p>Use condition</p> <p>Rearrange</p> <p>cao</p> <p>Use N2L, 3 terms, allow sign errors</p> <p>Multiply both sides by IF and express LHS as differential o.e.</p> <p>Rearrange</p> <p>Use condition</p> <p>cao</p> <p>Use N2L, 3 terms, allow sign errors</p> <p>Use condition</p>	
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	(vi)	$x = 12.25t + \frac{35.75}{-0.8} e^{-0.8t} + D$	M1	Integrate	
			A1	cao	
			M1	Use condition	
			A1	cao	
		Use $t = 0, x = 0: D = \frac{35.75}{0.8}$			
		When $t = 5$, distance = 105 m			
			[4]		

Question		Answer	Marks	Guidance
4.	(i)	$\ddot{x} = \dot{x} - \dot{y} - 3\sin t$	M1	Differentiate
		$\ddot{x} = \dot{x} - (5x - y - 12\sin t) - 3\sin t$	M1	Substitute for \dot{y}
		$y = x + 3\cos t - \dot{x}$	M1	Substitute for y
		$\ddot{x} + 4x = 9\sin t + 3\cos t$	A1	oe
		AE $m^2 + 4 = 0$	M1	
		$m = \pm 2j$	A1	
		CF $x = A\cos 2t + B\sin 2t$	F1	
		PI $x = P\sin t + Q\cos t$	B1	
		$\dot{x} = P\cos t - Q\sin t \quad \ddot{x} = -P\sin t - Q\cos t$	M1	Differentiate twice and substitute
		$3P = 9 \quad 3Q = 3$	M1	Equate coefficients and solve
		$P = 3, Q = 1$	M1	
		$x = A\cos 2t + B\sin 2t + 3\sin t + \cos t$	A1	cao
		[12]		
(ii)		$\dot{x} = -2A\sin 2t + 2B\cos 2t + 3\cos t - \sin t$	M1	Differentiate x
		Substitute x and \dot{x} in $y = x + 3\cos t - \dot{x}$	M1	
		$y = (A - 2B)\cos 2t + (2A + B)\sin 2t + 4\sin t + \cos t$	A1	
		[3]		

Question	Answer	Marks	Guidance
(iii)	$t = 0, y = 0: A - 2B + 1 = 0$ $t = 0, \frac{dy}{dt} = 5: 5 = 5A + 5 - (A - 2B) - 1$ Solve to give $A = 0, B = \frac{1}{2}$ $x = \frac{1}{2} \sin 2t + 3 \sin t + \cos t$ $y = -\cos 2t + \frac{1}{2} \sin 2t + 4 \sin t + \cos t$	M1 M1 A1 A1 [4]	Use condition Use condition
(iv)	$\cos 2t = \sin t$ $2 \sin^2 t + \sin t - 1 = 0: \sin t = \frac{1}{2} \quad \sin t = -1$ $t = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$ $\frac{2\pi}{3}$	M1 A1 M1 A1 A1 [5]	Equate their expressions for x and y cao Use double angle formula and attempt to solve quadratic equation One correct value cao

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4758 Differential Equations (Written Examination)

General Comments:

Candidates performed well on this paper and the majority of the responses were of a high standard. The level of accuracy displayed by most candidates was commendable. The methods required to solve the second order differential equations in Questions 1 and 4 were known by almost all candidates and these two questions were attempted by the majority of the candidates. Question 3 was the least popular question. Very few candidates attempted more than three questions.

Comments on Individual Questions:

Question No. 1

Second order linear differential equations

- (i) All candidates were familiar with the method of solution required in this part and there were very few arithmetical errors.
- (ii) The majority of the candidates applied the initial conditions accurately and scored full marks.
- (iii) Almost all candidates differentiated their solution from part (ii) and equated their derivative to zero. The resulting equation involved exponential terms and it was pleasing to see that most candidates knew how to solve this equation.
- (iv) Most candidates recognised the form of the particular integral that was required to solve this second differential equation and worked accurately to find the particular solution. A minority of the candidates did not use the product rule when differentiating the correct particular integral.

Question No. 2

First order differential equations

- (a)(i) Almost all candidates recognised that the given differential equation required the application of the integrating factor method and most began correctly by dividing through by x , the coefficient of $\frac{dy}{dx}$. Most candidates found the correct integrating factor and worked through accurately to find the general solution.
- (ii) All candidates applied the initial condition to their solution in part (i).
- (b)(i) Almost all candidates recognised that the given differential equation required the application of the separation of variables method. This led directly to a solution for y^{-1} . A common error when inverting their expression to find y was to invert term by term, rather than treating the expression as a whole.
- (ii) Candidates who found the maximum value of y by differentiation usually scored full marks. Those candidates who attempted to argue the result based on the possible values of the $\sin x$ that was present in the denominator of a fraction often produced incomplete solutions. A convincing explanation of how the maximum value of $\sin x$ led to the maximum value of y was required.

(c) Almost all candidates rearranged the given differential equation into the form required to apply Euler's method and many scored full marks. A minority of the candidates gave a list of numbers, none of which related to the correct ones, and it was not possible to award any marks. Sight of either 0.5403 or 0.537(2) or equivalent was required as evidence that the method was being applied correctly.

Question No. 3

First order differential equations

This was the least popular choice of question, but the candidates who opted for it almost always scored the majority of the marks.

- (i) This was a straightforward application of Newton's second law.
- (ii) This part required the application of the method of separation of variables resulting in a logarithmic expression involving v . The majority of the candidates worked accurately and found the given expression for v^2 in terms of x .
- (iii) Most candidates produced a correct sketch showing an increasing curve from the origin to a horizontal asymptote. The second mark was awarded for identifying the value of y corresponding to the asymptote.
- (iv) This was a simple numerical substitution from given information.
- (v) Most candidates were able to write down a correct differential equation obtained by applying Newton's second law. This differential equation could be solved by separating the variables, by using an integrating factor or by finding a complementary function and a particular integral. The latter was the most elegant approach and gave an expression for v in terms of t , without any further rearrangement being required. Most candidates separated the variables and errors often appeared in the subsequent algebraic manipulation.
- (vi) Most candidates integrated their answer to part (v), but many did not include a constant of integration.

Question No. 4

Simultaneous second order linear differential equations

- (i) There were many excellent responses to this part and the majority of the candidates scored full marks. There were occasional numerical slips when finding the coefficients in the particular integral.
- (ii) Almost all candidates gained the two method marks and the majority also gained the accuracy mark.
- (iii) All candidates made a good attempt at this part and most produced accurate solutions.
- (iv) Most candidates realised that they needed to equate their expressions for x and y . Those candidates who had worked accurately up to this point were able to simplify their expression to the trigonometric equation $\cos 2t = \sin t$. Solving this equation posed very few problems. Candidates who had been inaccurate in the earlier parts of the question were not usually able to solve the equation that they obtained by equating their expressions for x and y .

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u	
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	57	52	47	42	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	49	42	35	29	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	64	57	51	45	39	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	59	53	48	43	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	60	54	48	43	38	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	60	54	49	44	39	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	67	61	55	49	43	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	59	53	47	41	36	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	60	53	46	40	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	60	54	48	42	37	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	48	43	38	34	30	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	55	50	45	40	36	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	55	49	44	39	33	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0

UMS 100 80 70 60 50 40 0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u	
G241	01	Statistics 1 MEI (Z1)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u	
G244	01	Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1 MEI	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision 1 MEI	Raw	72	48	43	38	34	30	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2016 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering

			Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering	This unit has no entries in June 2016							
H860	02	Mathematics for Engineering								

Level 3 Certificate Mathematical Techniques and Applications for Engineers

			Max Mark	a*	a	b	c	d	e	u	
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u	
H866	01	Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H866	02	Critical maths	Raw	60	47	41	35	29	23	0
			Overall	132	111	96	81	66	51	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u	
H867	01	Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H867	02	Statistical problem solving	Raw	60	40	34	28	23	18	0
			Overall	132	103	88	73	59	45	0

Advanced Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u	
6993	01	Additional Mathematics	Raw	100	59	51	44	37	30	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u	
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0

Version	Details of change
1.1	Correction to Overall grade boundaries for H866
	Correction to Overall grade boundaries for H867